# SURFACE TENSION & VISCOSITY

## THEORY AND EXERCISE BOOKLET

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Syllabus:

Surface energy and surface tension, capillary rise; Viscosity (Poiseuille’s equation excluded), Stoke’s law; Terminal velocity
1.(a) **COHESIVE FORCE**

The force of attraction between the molecules of the same substance is called cohesive force.

In case of solids, the force of cohesion is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

**Example.**

(i) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

1.(b) **ADHESIVE FORCE**

The force at attraction between molecules of different substances is called adhesive force.

**Examples.**

(i) Adhesive force enables us to write on the black board with a chalk.

(ii) Adhesive force helps us to write on the paper with ink.

(iii) Large force of adhesion between cement and bricks helps us in construction work.

(iv) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

2. **SURFACE TENSION**

The property of a liquid at rest due to which its free surface tries to have minimum surface area and behaves as if it were under tension somewhat like a stretched elastic membrane is called surface tension.

The molecules of the liquid exert attractive forces on each other. There is zero net force on a molecule inside the volume of the liquid.

But a surface molecules is drawn into the volume. Thus, the liquid tends to minimize its surface area, just as a stretched membrane does.

Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid, the direction of this force being perpendicular to the line and tangential to the free surface of liquid. So if \( F \) is the force acting on one side of imaginary line of length \( L \) then

\[
T = \frac{F}{L}
\]

Regarding surface tension it is worth noting that:

(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.

(2) It is a scalar as it has a unique direction which is not to be specified.

(3) It has dimension \([\text{ML}^{-2}]\) and SI units N/m while CGS unit dyne/cm, so that one MKS unit of surface tension = \(10^2\) dyne/cm

(4) Surface tension of a liquid decreases with rise in temperature

(5) The surface tension of a liquid is very sensitive to impurities on the surface (called contamination) and decreases with contamination of surface.

(6) In case of soluble impurities surface tension may increase or decrease depending on the nature of impurity. Usually highly soluble salt such as sodium chloride increases surface tension while sparingly soluble salt such as soap decreases surface tension.
2.1 SURFACE ENERGY

When the surface area of a liquid is increased, the molecules from the interior rise to the surface. This requires work against force of attraction of the molecules just below the surface. This work is stored in the form of potential energy. Thus, the molecules in the surface have some additional energy due to their position. This additional energy per unit area of the surface is called 'surface energy'. The surface energy is related to the surface tension as discussed below:

Let a liquid film be formed on a wire frame and a straight wire of length l can slide on this wire frame as shown in figure. The film has two surfaces and both the surface are in contact with the sliding wire and hence, exert forces of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force TI. Thus, net force on the wire due to both the surface is 2TI. One has to apply an external force F equal and opposite to it to keep the wire in equilibrium. Thus,

\[ F = 2TI \]

Now, suppose the wire is moved through a small distance dx, the work done by the force is,

\[ dW = F \, dx = (2TI) \, dx \]

But \((2l) \, (dx)\) is the total increase in area of both the surface of the film. Let is be dA. Then,

\[ dW = T \, dA \]

or

\[ T = \frac{dW}{dA} \]

Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity.

Ex.1 Calculate the energy released when 1000 small water drops each of same radius \(10^{-7}\) m coalesce to form one large drop. The surface tension of water is \(7.0 \times 10^{-2}\) N/m.

Sol. Let \(r\) be the radius of smaller drops and \(R\) of bigger one. Equating the initial and final volumes, we have

\[
\frac{4}{3} \pi R^3 = (1000) \left( \frac{4}{3} \pi r^3 \right)
\]

or

\[ R = 10 \, r = (10)(10^{-7}) \, \text{m} \quad \text{or} \quad R = 10^{-6} \, \text{m} \]

Further, the water drops have only one free surface. Therefore,

\[ \Delta A = 4\pi R^2 - (1000)(4\pi r^2) \]

\[ = 4\pi \left[ (10^{-6})^3 - (10^{-7})^3 \right] = 36\pi \times 10^{-12} \, \text{m}^2 \]

Here, negative sign implies that surface area is decreasing. Hence, energy released in the process.

\[ U = T |\Delta A| = (7 \times 10^{-2})(36\pi \times 10^{-12}) \, \text{J} = 7.9 \times 10^{-12} \, \text{J} \]

Ans.

Ex.2 A mercury drop of radius 1 cm is sprayed into \(10^6\) droplets of equal size. Calculates the energy expanded if surface tension of mercury is \(35 \times 10^{-3}\) N/m.

Sol. If drop of radius \(R\) is sprayed into \(n\) droplets of equal radius \(r\), then as a drop has only surface, the initial surface area will be \(4\pi R^2\) while final area is \(n \times (4\pi r^2)\). So the increase in area

\[ \Delta S = n(4\pi r^2) - 4\pi R^2 \]

So energy expended in the process,

\[ W = T \Delta S = 4\pi T \left[ n r^2 - R^2 \right] \]

Now since the total volume of \(n\) droplets is the same as that of initial drop, i.e.,

\[ \frac{4}{3} \pi R^3 = n [(4/3)\pi r^3] \quad \text{or} \quad r = \frac{R}{n^{1/3}} \]

Putting the value of \(r\) from equation (2) in (1)

\[ W = 4\pi R^2 T \left( n^{1/3} - 1 \right) \]
2.2 Excess Pressure Inside a liquid drop:
Consider a liquid drop of radius 'R' and surface tension 'T'. A liquid drop has only one surface film, hence the surface tension force is $T(2\pi R)$.
Force due to inside pressure ($P_{in}$) is $P_{in} \times \text{area}$ i.e. $P_{in} \pi R^2$
similarly force due to outside pressure ($P_0$) is $P_0 \pi R^2$ since each half of the liquid drop is in equilibrium

$P_0 \pi R^2 + T(2\pi R) = P_{in}(\pi R^2)$

$P_{in} - P_0 = \frac{2T}{R}$ = Excess Pressure

2.3 Excess pressure inside a bubble
Consider a bubble of radius 'R' and surface tension 'T'. A bubble consists of two spherical surface films with a thin layer of liquid between them.
The total surface tension force for each surface inner and outer $T(2\pi R)$ for a total of $(2T)(2\pi R)$
Force due to inside pressure ($P_{in}$) is $P_{in} \pi R^2$ and due to outside pressure ($P_0$) is $P_0 \pi R^2$

Since each half of bubble is in equilibrium (lower half shown in figure)

$P_0 \pi R^2 + 2T(2\pi R) = P_{in} \pi R^2$

$P_{in} - P_0 = \frac{4T}{R}$ = Excess pressure

Note: (1) If we have an air bubble inside a liquid, a single surface is formed. There is air on the concave side and liquid on the convex side. The pressure in the concave side (that is in the air) is greater than the pressure in the convex side (that is in the liquid) by an amount $\frac{2T}{R}$.

$\therefore P_2 - P_1 = \frac{2T}{R}$

The above expression has been written by assuming $P_1$ to be constat from all sides of the bubble. For small size bubbles this can be assumed.

(2) From the above discussion, we can make a general statement. The pressure on the concave side of a spherical liquid surface is greater than the convex side by $\frac{2T}{R}$.

3. For any curved surface excess pressure on the concave side = $T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ where $R_1$ & $R_2$ are radius of curvature of the surface in two perpendicular direction of instead of liquid surface, liquid film is given then above expression will be

$P = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ For spherical curved surface $R_1, R_2$
Ex.3 What should be the pressure inside a small air bubble of 0.1 mm radius situated just below the water surface. Surface tension of water \(7.2 \times 10^{-2} \text{N/m}\) and atmospheric pressure \(1.013 \times 10^5 \text{N/m}^2\).

**Sol.**

Surface tension of water \(T = 7.2 \times 10^{-2} \text{N/m}\)

Radius of air bubble \(R = 0.1 \text{ mm} = 10^{-4} \text{ m}\)

The excess pressure inside the air bubble is given by,

\[
P_2 - P_1 = \frac{2T}{R}
\]

:. Pressure inside the air bubble,

\[
P_2 = P_1 + \frac{2T}{R}
\]

Substituting the values, we have

\[
P_r = (1.013 \times 10^5) + \frac{2 \times 7.2 \times 10^{-2}}{10^{-4}} = 1.027 \times 10^5 \text{N/m}^2
\]

Ex.4 A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76 cm of mercury.

**Sol.**

The total pressure inside the bubble at depth \(h_1\) is \((P \text{ is atmospheric pressure})

\[
(P + h_1 \rho g) + \frac{2T}{r_1} = P_r
\]

and the total pressure inside the bubble at depth \(h_2\) is \((P + h_2 \rho g) + \frac{2T}{r_2} = P_r

Now, according to Boyle's Law

\[
P_1 V_1 = P_2 V_2
\]

where \(V_1 = \frac{4}{3} \pi r_1^3\), and \(V_2 = \frac{4}{3} \pi r_2^3

Hence we get

\[
\left[(P + h_1 \rho g) + \frac{2T}{r_1}\right] \frac{4}{3} \pi r_1^3 = \left[(P + h_2 \rho g) + \frac{2T}{r_2}\right] \frac{4}{3} \pi r_2^3
\]

or,

\[
\left[(P + h_1 \rho g) + \frac{2T}{r_1}\right] r_1^3 = \left[(P + h_2 \rho g) + \frac{2T}{r_2}\right] r_2^3
\]

Given that \(h_1 = 100 \text{ cm}, r_1 = 0.1 \text{ mm} = 0.01 \text{ cm}, r_2 = 0.126 \text{ mm} = 0.0126 \text{ cm}, T = 567 \text{ dyne/cm}, P = 76 \text{ cm of mercury}\). Substituting all the values, we get

\[
h_2 = 9.48 \text{ cm}
\]

2.4 Pressure inside a charged bubble

Consider a charged bubble of radius 'R', surface tension 'T' and surface charge density \(\sigma\)

The total surface tension force for each surface (inner and outer) is \(T \times (2\pi R)\) for a total of \(2T \times (2\pi R)\)

Force due to inside pressure \((P_{in})\) is \(P_{in} \pi R^2\) and due to outside pressure \((P_0)\) is \(P_0 \pi R^2\)

\[
2T \times (2\pi R) + P_{o} \pi R^2 = P_{in} \pi R^2 + \frac{\sigma^2}{2\varepsilon_0} \pi R^2
\]

\[
P_{in} = P_0 + \frac{4T}{R} - \frac{\sigma^2}{2\varepsilon_0}
\]
3. CONTACT ANGLE AND SHAPE OF LIQUID SURFACE

The surface of a liquid when meets a solid, such as the wall of a container, it usually curves up or down near the solid surface. The angle which the tangent to the is called the contact angle. The curved liquid surface at the pt. of surface of the liquid is called meniscus. The shape of the meniscus contact of liquid surface with (convex or concave) is determined by the relative strengths of solid cohesive and adhesive forces surface with the solid surface inside the liquid.

When the adhesive force (P) between solid and liquid molecules is more than the cohesive force (Q) between liquid-liquid molecules (as with water and glass), shape of the meniscus is concave and the angle of contact $\theta$ is less than $90^\circ$. In this case the liquid wets or adheres to the solid surface. The resultant (R) of P and Q passes through the solid.

On the other hand when $P < Q$ (as with glass and mercury), shape of the meniscus is convex and the angle of contact $\theta > 90^\circ$. The resultant (R) of P and Q in this case passes through the liquid.

Let us now see why the liquid surface bends near the contact with a solid. A liquid in equilibrium can not sustain tangential stress. The resultant force on any small part of the surface layer must be perpendicular to the surface at that point. Basically three forces are acting on a small part of the liquid surface near its contact with solid. These forces are:

(i) $P$, attraction due to the molecule of the solid surface near it i.e. adhesive force which acts outwards at right angle to the wall of tube.

(ii) $Q$, attraction due to liquid molecules near this part and i.e. cohesive force which acts at an angle of $45^\circ$ to the vertical.

We have considered very small part, so weight of that part can be ignored for better understanding. As we have seen in the last figures, to make the resultant (R) of P and Q perpendicular to the liquid surface the surface becomes curved (convex or concave).

Note: The angle of contact between water and clean glass is zero.

4. CAPILLARY RISE

If a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarily.

In order to calculate the height to which a liquid will rise in a capillary, consider a glass capillary of radius R dipped in water as shown in Fig. shown. As the meniscus is concave and nearly spherical, the pressure below the meniscus will be $[p_0 - (2T/r)]$ with $p_0$ as atmospheric pressure and r as radius of meniscus. Now as liquid flows from higher to lower pressure and at same level in a liquid pressure must be same (this is because a liquid cannot sustain tangential stress), so the liquid will ascend in the capillary till hydrostatic pressure of the liquid compensates for the decrease in pressure. i.e.,
\[ p_0 = \left[ p_0 - \frac{2T}{r} \right] + h \rho g \quad \text{or} \quad h = \frac{2T}{r \rho g} \quad \ldots(1) \]

But from figure shown it is clear that radius of meniscus \( r \) is related to the radius of capillary through the relation
\[ \frac{R}{r} = \cos \theta, \quad \text{i.e.,} \quad r = \frac{R}{\cos \theta} \quad \ldots(2) \]
where \( \theta \) is the angle of contact. *So substituting the value of \( r \) from Eqn. (2) in (1), we get
\[ h = \frac{2T}{r \rho g} = \frac{2T \cos \theta}{R \rho g} \quad \ldots(3) \]

**Alternate Method**

As it can be seen from figure that \( T \sin \theta \) cancels out:
The force due to \( T \cos \theta \) balances the weight of liquid \( (mg = \rho \nu g) \)
vol. of the curve is negligible
\[ \therefore \text{vol. of liquid in } \pi^2 h \]

\[ T \cos \theta = 2\pi r = \pi^2 hg \Rightarrow h = \frac{2T \cos \theta}{r \rho g} \]

This is the desired result and from this it is clear that:

1. The capillarity depends on the nature of liquid and solid both, i.e., on \( T, \rho, \theta \) and \( R \). If \( \theta > 90^\circ \), i.e., meniscus is convex, \( h \) will be negative, i.e., the liquid will descend in the capillary as actually happens in case of mercury in a glass tube. However, if \( \theta = 90^\circ \), i.e., meniscus is plane, \( h = 0 \) and so no capillarity.
(2) For a given liquid and solid at a given place as \( \rho, T, \theta \) and \( g \) are constant, (figure shown)
\[ h_r = \text{constant} \]
\[ \therefore \] lesser the radius of capillary greater will be the rise and vice-versa. (figure shown)

(3) Here it is important to note that in equilibrium the height \( h \) is independent of the shape of capillary if the radius of meniscus remains the same. This is why the vertical height \( h \) of a liquid column in capillaries of different shapes and sizes will be same if the radius of meniscus remains the same and also the vertical height of the liquid in a capillary does not change, when it is inclined to the vertical. (figure shown)

4. Capillarity has large number of applications in our daily life, e.g.,
(a) The oil in the wick of a lamp rise due to capillary action of threads in the wick.
(b) Action of towel in soaking up moisture from the body is due to capillary action of cotton in the towel.
(c) Water is retained in a piece of sponge on account of capillarity.
(d) A blotting paper soaks ink by capillary action of the pores in the blotting paper.
(e) The root-hairs of plants drawn water from the soil through capillary action.

5. In Case of glass and water \( \theta = 0^\circ \)
here force due to surface tension balances the weight of the liquid \( (\rho \times V \times g) \)

volume of the liquid = \( \pi r^2 h + \frac{2}{3} \pi r^3 \)

where \( \pi r^3 - \frac{2}{3} \pi r^3 \) is the volume of the curve which is not negligible in this case

\[ \therefore 2\pi r = \rho (\pi r^2 h + \frac{2}{3} \pi r^3) g \]

\[ 2T = rh \rho g + \frac{1}{3} \pi r^2 \rho g \]

6. If two parallel plates with the spacing \( d \) are placed in water reservoir, then height or rise.

\[ 2T \ell = \rho \ell d g \]

\[ h = \frac{2T}{\rho \ell g} \]

7. If two concentric tubes of radius \( r_1 \) and \( r_2 \) (inner one is solid) are placed in water reservoir, then height of rise?

\[ T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h] \rho g \]

\[ h = \frac{2T}{(r_2 - r_1) \rho g} \]
8. If weight of the liquid in the meniscus is to be consider:

\[ T \cos \theta \times 2\pi r = \left[ \pi r^2 h + \frac{1}{3} \pi r^2 \times r \right] \rho g \]

\[ h + \frac{r}{3} = \frac{2T \cos \theta}{\rho g} \]

9. When capillary tube (radius, 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by \( p_1 = \frac{2T}{R_1} \)

where \( R_1 = \) radius of curvature of upper meniscus.

The hydrostatic pressure \( p_2 = h \rho g \) is always directed downwards.

If \( p_1 > p_2 \), i.e. resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward (fig a). The radius of lower meniscus \( R_2 \) can be given by

\[ \frac{2T}{R_2} = (p_1 - p_2) \]

If \( p_1 < p_2 \), i.e. resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (fig b)

The radius of lower meniscus can be given by \( \frac{2T}{R_2} = p_2 - p_1 \)

If \( p_1 = p_2 \), then no resulting pressure. then, \( p_1 - p_2 = \frac{2T}{R_2} = 0 \) or, \( R_2 = \infty \) i.e. lower surface will be FLAT (fig c)

Ex.5 A drop of water volume 0.05 cm\(^3\) is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm\(^2\). If the surface tension of water is 70 dyne/cm, find the normal force required to separate out the two glass plates in newton.

Sol. Pressure inside the film is less than outside by an amount,

\[ P = T \left[ \frac{1}{r_1} + \frac{1}{r_2} \right] \]

where \( r_1 \) and \( r_2 \) are the radii of curvature of the meniscus. Here \( r_1 = \frac{t}{2} \) and \( r_2 = \infty \), then the force required to separate the two glass plates, between which a liquid film is enclosed (figure) is, \( F = P \times A = \frac{2AT}{t} \), where \( t \) is the thickness of the film, \( A = \) area of film.

\[
F = \frac{2A^2T}{At} = \frac{2A^2T}{V} = \frac{2 \times (40 \times 10^{-4})^2 \times (70 \times 10^{-2})}{0.05 \times 10^{-4}} = 45 \text{ N}
\]
4.1 CAPILLARY RISE IN A TUBE OF INSUFFICIENT HEIGHT

We know, the height through which a liquid rises in the capillary tube of radius \( r \) is given by

\[
\therefore h = \frac{2T}{\rho g} \quad \text{or} \quad h R = \frac{2T}{\rho g} = \text{constant}
\]

When the capillary tube is cut an its length is less then \( h \) (i.e. \( h' \)), then the liquid rises upto the top of the tube and spreads in such a way that the radius (\( R' \)) of the liquid meniscus increases and it becomes more flat so that \( hR = h'R' = \text{Constant} \). Hence the liquid does not overflow.

If \( h' < h \) then \( R' > R \) or \( \frac{r}{\cos \theta'} > \frac{r}{\cos \theta} \Rightarrow \theta' > \theta \)

5. VISCOSITY AND NEWTON'S LAW OF VISCOUS FORCE

In case of steady flow of a fluid when a layer of fluid slips or tends to slip on adjacent layer in contact, the two layers exert tangential force on each other which tries to destroy the relative motion between them. The property of a fluid due to which it opposes the relative motion between its different layers is called viscosity (or fluid friction or internal friction) and the force between the layers opposing the relative motion viscous force. A briskly stirred fluid comes to rest after a short while because of viscosity.

As a result of large number of experiments Newton found that viscous force \( F \) acting on any layer of a fluid is directly proportional to its area \( A \) and to the velocity gradient \((dv/dy)^*\) at the layer i.e.,

\[
F \propto A \frac{dv}{dy} \quad \text{or} \quad F = -\eta A \frac{dv}{dy} \quad ...(1)
\]

when \( \eta \) is a constant called coefficient of viscosity or simply viscosity of the fluid. The negative sign shows that viscous force on a liquid layer acts in a direction opposite to the relative velocity of flow of fluid. The Eq. (1) is known as Newton's law of viscous force. Here \( y \) is taken from the layer of which velocity is zero.

Regarding viscosity of fluid it is worth noting that:

1. It depends only on the nature of fluid and is independent of area considered or velocity gradient.
2. Its dimensions are \([ML^{-1}T^{-1}]\) and SI unit poiseuille (PI) while CGS unit dyne-s/cm\(^2\) called poise (P) with
   \[
   1 \text{ PI} = 10 \text{ poise}
   \]
3. Viscosity of liquids is much greater (say about 100 times more) than that of gases
   i.e., \( \eta_L > \eta_g \)
Ex. 6  A boat of area $10 \, \text{m}^2$ floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential water in contact with the bed is stationary, find the tangential force needed to keep the boat moving with same velocity. Viscosity of water is 0.01 poise.

Sol.  As velocity changes from 2 m/s at the surface to zero at the bed which is at a depth of 1 m.

Velocity gradient $= \frac{dv}{dy} = \frac{2 - 0}{1} = 2 \, \text{s}^{-1}$

Now from Newton's law of viscous force,

$$|F| = \eta A \frac{dv}{dy} = (10^{-2} \times 10^{-1}) \times 10 \times 2 = 0.02 \, \text{N}$$

Ex. 7  The velocity of water in a river is 18 km/hr at the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The viscosity of water is $10^{-3}$ poiseuille.

Sol.  As velocity at the bottom of the river will be zero, velocity gradient

$$\frac{dv}{dy} = \frac{18 \times 10^3}{60 \times 60 \times 5} = 1 \, \text{s}^{-1}$$

Now as the viscous force $F = \eta A \frac{dv}{dy}$ is tangential to the area,

Shear stress $= \frac{F}{A} = \eta \frac{dv}{dy} = 10^{-3} \times 1 = 1 \times 10^{-2} \, \text{N/m}^2$

Ex. 8  A cylinder of mass radius $r_1$ and length $\ell$ is kept inside another cylinder of radius $r_2$ and length $\ell$. The space between them is filled with a liquid of viscosity $\eta$. The inner cylinder starts rotating with angular velocity $\omega$ while the other cylinder is at rest. Find time when inner cylinder stops.

Sol.  Viscous force $F = - \eta A \frac{dv}{dy}

$$= - \eta 2\pi r_1 \frac{\phi r_1^2}{r_2 - r_1} = -\eta 2\pi \frac{\phi r_1^2}{r_2 - r_1}$$

$$\tau = |F \times \vec{r}_1| = Fr_1 \sin 90^\circ = Fr_1 = \eta 2\pi \frac{\phi r_1^2}{r_2 - r_1}$$

$$\tau = I \alpha = \frac{M r_1^2}{2} \frac{d\omega}{dt} = -\eta 2\pi \frac{\phi r_1^2}{r_2 - r_1}$$

from eq. (1)

$$\frac{a r_1^2}{M (r_2 - r_1)} \int_0^\omega d\omega = -\int_0^{\omega_0} d\omega$$

$$\frac{a r_1^2}{M (r_2 - r_1)} t = \eta \omega_0$$
6. **STOKES LAW**

When a body moves through a fluid, the fluid in contact with the body is dragged with it. This establishes relative motion in fluid layers near the body, due to which viscous force starts operating. The fluid exerts viscous force on the body to oppose its motion. The magnitude of the viscous force depends on the shape and size of the body, its speed and the viscosity of the fluid. Stokes established that if a sphere of radius \( r \) moves with velocity \( v \) through a fluid of viscosity \( \eta \), the viscous force opposing the motion of the sphere is

\[
F = 6 \pi \eta rv
\]

7. **TERMINAL VELOCITY (\( v_T \))**

Consider a small sphere falling from rest through a large column of viscous fluid. The forces acting on the sphere are,

(i) Weight \( W \) of the sphere acting vertically downwards
(ii) Upthrust \( F_u \) acting vertically upwards
(iii) Viscous force \( F_v \) acting vertically upwards, i.e., in a direction opposite to velocity of the sphere.

Initially, \( F_v = 0 \) and \( W > F_u \)

and the sphere accelerates downwards. As the velocity of the sphere increases, \( F_v \) increases, eventually a stage is reached when \( W = F_u + F_v \). After this net force on the sphere is zero and it moves downwards with a constant velocity called terminal velocity (\( v_T \)).

Substituting proper values in Eq. (i) we have,

\[
\frac{4}{3} \pi r^3 \rho g = \frac{4}{3} \pi r^3 \sigma g + 6 \pi \eta rv
\]

Here, \( \rho = \) density of sphere, \( \sigma = \) density of fluid and \( \eta = \) coefficient of viscosity of fluid.

From Eq. (ii), we get

\[
v_T = \frac{2}{9} \left( \frac{r^2}{\rho - \sigma} \right) g
\]

Figure shows the variation of the velocity \( v \) of the sphere with time.

**Note:** From the above expression we can see that terminal velocity of a spherical body is directly proportional to the difference in the densities of the body and the fluid (\( \rho - \sigma \)). If the density of fluid is greater than that of body (i.e., \( \sigma > \rho \)), the terminal velocity is negative. This means that the body instead of falling, moves upward. This is why air bubbles rise up in water.

**Ex.9** Two spherical raindrops of equal size are falling vertically through air with a terminal velocity of 1 m/s. What would be the terminal speed if these two drops were to coalesce to form a large spherical drop?

**Sol.**

\( v_T \propto r^2 \)

Let \( r \) be the radius of small raindrops and \( R \) the radius of large drop. Equating the volumes, we have

\[
\frac{4}{3} \pi r^3 = 2 \left( \frac{4}{3} \pi \right)
\]

\[
\therefore \quad R = (2)^{1/3}, \quad \text{or} \quad \frac{R}{r} = (2)^{1/3}
\]

\[
\therefore \quad \frac{v_T'}{v_T} = (\frac{r}{R})^2 = (2)^{2/3}
\]

\[
\therefore \quad v_T' = (2)^{2/3} v_T = (2)^{2/3}(1.0) \text{ m/s} = 1.587 \text{ m/s} \quad \text{Ans.}
\]